

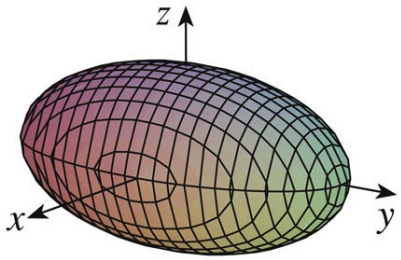
Lesson 10. Cylinders and Quadric Surfaces, cont.

1 Today...

- A special family of surfaces in 3D space

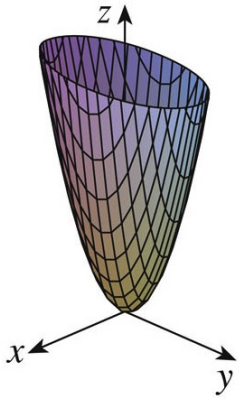
2 Quadric surfaces

- **Ellipsoid**



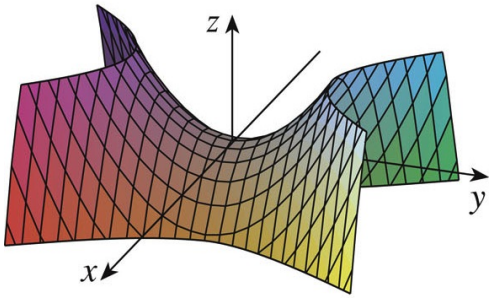
  - Equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
  - All traces are ellipses
  - If $a = b = c$, the ellipsoid is a sphere

- **Elliptic paraboloid**



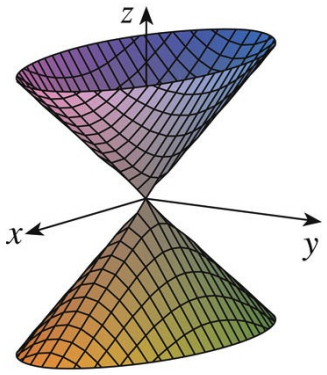
  - Equation: $\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$
  - Horizontal traces are ellipses
  - Vertical traces are parabolas
  - The variable raised to the first power indicates the axis of the paraboloid

- **Hyperbolic paraboloid**



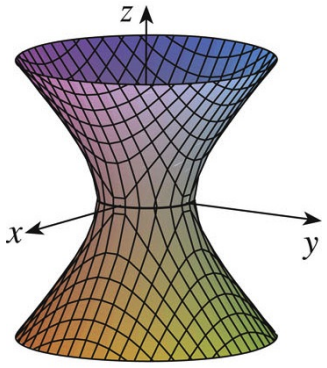
  - Equation: $\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$
  - Horizontal traces are hyperbolas
  - Vertical traces are parabolas
  - The case when $c < 0$ is illustrated

- **Cone**



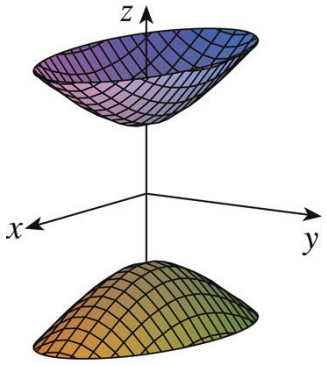
  - Equation: $\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$
  - Horizontal traces are ellipses
  - Vertical traces are planes or hyperbolas

- **Hyperboloid of one sheet**



  - Equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
  - Horizontal traces are ellipses
  - Vertical traces are hyperbolas

- **Hyperboloid of two sheets**

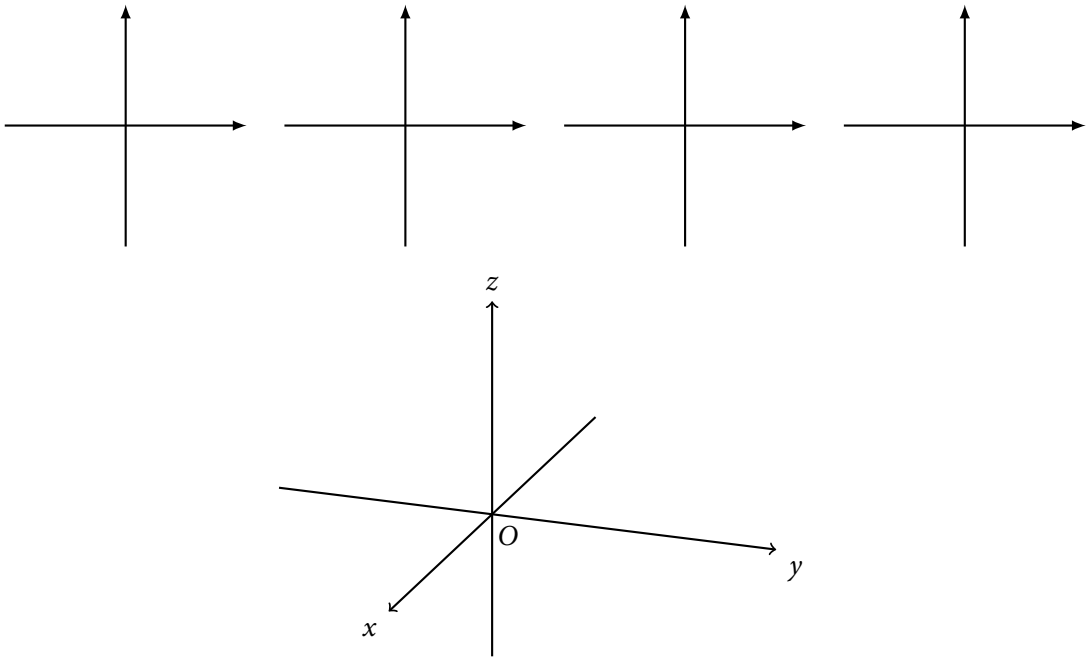


  - Equation: $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
  - Horizontal traces (when $z = k$) are ellipses if $k > c$ or $k < -c$
  - Vertical traces are hyperbolas

- Equations given above are in "standard form"
  - May need to do some algebra to get an equation into standard form
- Equations given above are for surfaces that are symmetric about the z-axis
  - May need to switch the variables around to match an equation with the surface type

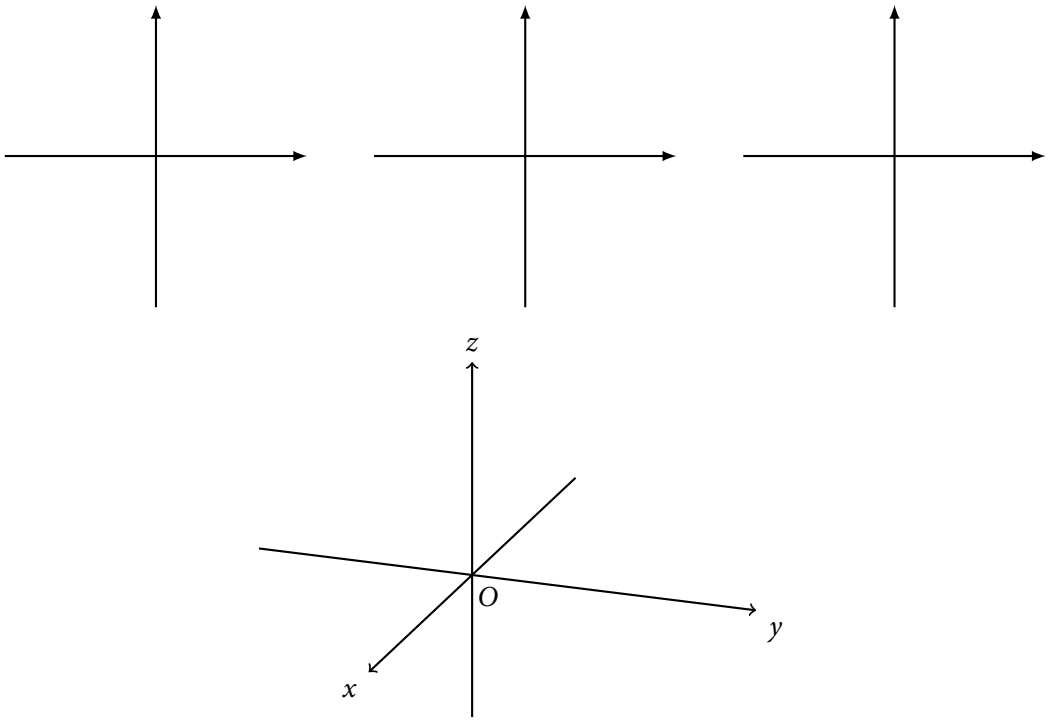
Example 1. Sketch the quadric surface $z = x^2 - \frac{y^2}{4}$. What is this quadric surface called?

Hint. Draw traces for this surface when $y = 0$, $x = 0$, $x = 1$, and $x = -1$.

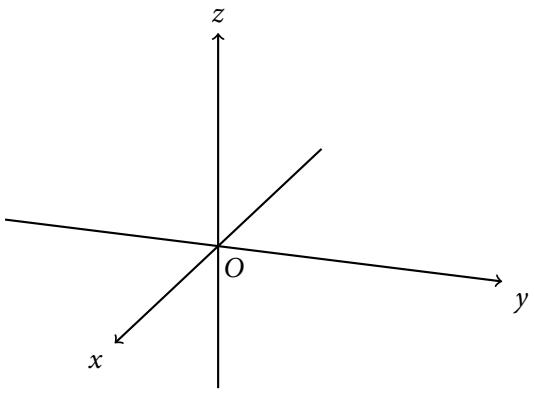


Example 2. Sketch the quadric surface $x^2 + y^2 - z^2 = 1$. What is this quadric surface called?

Hint. Draw traces for this surface when $z = 0$, $x = 0$, and $y = 0$.



Example 3. Identify and sketch the quadric surface $4x^2 + y^2 - 2z^2 + 4 = 0$ by matching the equation to the standard equations given above.



Example 4. Identify and sketch the quadric surface $2y^2 = x^2 + 4z^2$ by matching the equation to the standard equations given above.